The Traveling Salesman Problem with positional consistency constraints

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Abstract. The Travelling Salesman Problem (TSP) with positional consistency constraints is defined as follows. Consider a complete graph, $G = (V, A)$, with $V = \{1, ..., n\}$ being the set of nodes, and $A = \{(i, j) : i, j \in V, i \neq j\}$ being the set of arcs. Node 1 is the depot, and nodes $2, ..., n$ are clients that require one or more services. Additionally, a set of m routes must be generated, and it is known *a priori* which clients must be visited at each route. Finally, all the clients that require more than one service also require consistency, which means that they must be visited in the same position in all the routes they appear in. The aim is to generate a minimum cost set of routes, all of them starting and ending at the depot, while ensuring that all the nodes are visited in all the routes they require service in, and consistency constraints are verified.

This type of consistency is relevant in scheduling applications, where each route represents a schedule, which is divided in fixed homogeneous time slots. Assigning a node to a relative position in a route corresponds to assigning a client to a time slot in the schedule. Consistency is enforced by ensuring that each node appears in exactly the same position/time slot in all the routes it must be served in.

Also, idling is not allowed between two client nodes, but if different routes have different sizes (measured by the number of nodes in a route), smaller routes are allowed to start later than the largest route, if this results in an improvement in the total cost of the solution.

Positional consistency appears as an alternative to temporal consistency, where travel times (and eventually service times) are incorporated as parameters and used to write constraints where an upper limit is imposed to the difference between arrival times for the same client in different routes. The main difference between the two approaches lies in the fact that, while in positional consistency the schedule is divided in fixed time slots, in temporal consistency a node can be visited at any moment of the schedule. Additionally, whereas constraints that state that a client must be assigned to the same position in all the routes it appears in are adequate in positional consistency, temporal consistency constraints with tight upper bounds may be impractical. Several works addressing temporal consistency can be found in the literature (see, for example, [7] and [8]).

Since the position that a node occupies in a route is important for positional consistency constraints, several time dependent models known from the literature (see [3], [4] and [6]) were adapted for this problem. These models use binary variables, z_{ij}^{kl}, that take value 1 if arc (i, j) is traversed in position k in route l, and 0 otherwise. These "position-indexed" variables allow us to write several sets of consistency constraints, some of them being relatively strong. However, it is also true that, for these models, the number of decision variables and constraints grows quite quickly when the dimension of the problem, namely the number of routes, increases.
As a motivation for one of the main contributions of our work, we observe that there still exist routing problem variants for which formulations that do not use route-index variables (that is, an index l identifying the route) have not yet been described, and we believe that it is unlikely that such models can be developed. Two such variants are the Split Delivery Vehicle Routing Problem (see, for example, [1]) and the Arc Routing Problem, (see, for instance, [5]). Also, routing problems involving several routes and synchronisation constraints are, in general, difficult to model without route-indexed variables (see, for instance, [2]). However, the characteristics of the problem we study allow us to propose a valid and aggregated version of one of the time-dependent models, that uses variables u_{ij}^{k} that are not indexed by route and represent the number of routes that traverse an arc (i, j) in position k.
Our computational results show that in many cases the new model provides a better computational performance than the other models in this study. A more complete computational experiment was also carried out to assess and compare the models under study. Several consistency configurations were considered, in order to observe how the number of nodes requiring consistency, as well as the number of routes those nodes must appear in, affects the performance of these models.

Keywords: combinatorial optimization, Traveling Salesman Problem, positional consistency

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