A new Exact-Subgraph-Based Hierarchy for Stable Set

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Abstract. One of several hierarchies towards the stability number of G is the exact subgraph hierarchy (ESH). On the first level it computes the Lovász theta function $\vartheta(G)$ as semidefinite program (SDP) with a matrix variable of order $n+1$ and $n+m+1$ constraints. On the k-th level it adds all exact subgraph constraints (ESC) for subgraphs of order k to the SDP. An ESC ensures that the submatrix of the matrix variable corresponding to the subgraph is in the correct polytope. By including only some ESCs into the SDP the ESH can be exploited computationally. We introduce a variant of the ESH that computes $\vartheta(G)$ through an SDP with a matrix variable of order n and $m+1$ constraints. We show that it makes sense to include the ESCs into this SDP and introduce the compressed ESH (CESH) analogously to the ESH. Computationally the CESH seems favorable as the SDP is smaller. However, we prove that the bounds based on the ESH are always at least as good as those of the CESH. In computational experiments sometimes they are significantly better.
We also introduce scaled ESCs (SESCs), which are a more natural way to include exactness constraints into the smaller SDP and we prove that including an SESC is equivalent to including an ESC for every subgraph.

Keywords: Semidefinite programming, Relaxation hierarchy, Stable set, Exact subgraph constraints