An Integer Linear Programming approach to the Time-constrained Vehicle Routing Problem with Time-windows

Maria João Lopes[1] and Maria Teresa Godinho[2]

[1] Department of Quantitative Methods, Iscte-IUL University Institue of Lisbon and CMAFcIO, University of Lisbon, Lisbon, Portugal
`maria.joao.lopes@iscte-iul.pt`,
[2] Department of Mathematics and Physical Sciences, Polytechnic Institute of Beja, Beja, Portugal, and CMAFcIO, University of Lisbon, Lisbon, Portugal

Abstract. We study a problem arising in home healthcare nursing. Organizing home visits to patients involves features of various OR problems, namely scheduling, rostering and routing. In this work, we address the routing problem, modeled as a Time-constrained Vehicle Routing Problem with Time Windows (TCVRPTW). An integer linear programming formulation of the TCVRPTW is introduced. Preliminary computational results will be presented at the talk.

Keywords: Home HeathCare, Time-constrained Vehicle Routing Problem with Time-windows, Integer Linear Programming formulations

1 Introduction

In the last few years, the healthcare sector has witnessed several paradigm shifts. With the increase in the human life span, the number of people requiring long term specialised healthcare is rising, but hospitalizations are costly and risky. In such an environment, home healthcare is a promising solution and a growing trend.

In this work, we address a routing problem problem related to a Residential Long-term Care Unit, located in Beja, Portugal. The Unit is made up of multidisciplinary healthcare teams that pay visits to the patients in their homes. Due to medical reasons or self convenience of the patients, some of the visits have to be payed in given time-slots. According to whether or not this need exists, the care to be provided is classified as *specific* or as *general*. On each shift, each team is assigned with a group of patients, grouped and sequenced according to the proximity of their homes and time compatibility. The team uses a vehicle to travel to and between patients' homes. The itinerary is left to the driver to decide. The problem we just described concerns both scheduling and routing issues. This work, however, deals only with the routing part of the problem.

2 Problem Definition and ILP model

Consider the set $Pat = \{1, ..., P\}$ of the patients to treat and the set $C = \{s, g\}$ of the two types of care (specific and general) the Unit can provide. Each pair, p^c, defines a task to be executed. Observe that p^s and p'^g with $p' = p$ may coexist, as patients may need the two types of care. In this case, there will be two tasks associated with the same patient. Any team can perform any task, but the tasks type s (that is the tasks associated with a care type s) have to be executed within a specific time window, whereas the tasks type g (that is the tasks associated with a care type g) have no time restrictions.

Let $G = (N, A)$ be a complete directed graph with $N = \{0, ..., n\}$. Node 0 represents the depot and the remaining nodes represent the tasks. We let N^s and N^g denote the subsets of the nodes associated with tasks type s and g, respectively. Two attributes are associated with the nodes: d_i indicates the time required to complete the task to be executed in $i \in N \backslash \{0\}$ and $[a_i, b_i]$ denotes the time window within the task associated with $i \in N^s$ has to be concluded. The arcs represent the trips. Two attributes are associated with each arc $(i, j) \in A$: t_{ij} indicates the time needed to travel from the home of the patient associated with the task represented by i to the home of the patient associated with the task represented by j and c_{ij} that indicates its cost. If nodes i and j are associated with the same patient, t_{ij} and c_{ij} are null. Finally, we denote by T the maximum duration of a route and by c_v the cost of using a vehicle. The fleet has size V.

The Time-constrained Vehicle Routing Problem with Time-windows (TCVRPTW) seeks a set of routes of minimal total cost in the defined network, such as time-windows and the number of vehicles and duration of the routes requirements are observed.

Next, an integer linear programming formulation of the TCVRPTW is introduced. For simplicity, we resort to p^c notation to designate a node when it is important to specify the pair associated with the task it represents. We assume that all parameters are non-negative integers (the time unit is minutes).

Consider the decision variables:

- binary variables, $x_{ij} = 1$ if j is visited just after i; $x_{ij} = 0$, otherwise, for all $i, j \in N, i \neq j$;
- variables y_{ij} representing the arrival time at j just after visiting i, for all $i, j \in N, i \neq j$;
- w_{ij} representing the waiting time at j, after travelling from i to j, for all $i \in N, i \neq j$. (note that $w_{ij} = 0$, for $j \in N^g \cup \{0\}$)
- binary variables $u_{p^s,p'^g} = 1$, if care s is provided before care g to a patient requiring both types of care, $u_{p^s,p'^g} = 0$, otherwise, for all $p^s \in N^s$, $p'^g \in N^g$, with $p = p'$.

We have the formulation:

$$\min \; Z = \sum_{j \in N \setminus \{0\}} c_v x_{0j} + \sum_{i,j \in N, i \neq j} c_{ij} x_{ij} \tag{1a}$$

subject to

$$\sum_{i \in N} x_{ij} = 1 \qquad j \in N \setminus \{0\} \tag{1b}$$

$$\sum_{j \in N} x_{ij} = 1 \qquad i \in N \setminus \{0\} \tag{1c}$$

$$y_{0j} = t_{0j} x_{0j} \qquad j \in N \setminus \{0\} \tag{1d}$$

$$\sum_{i \in N} (y_{ij} + w_{ij} + d_j x_{ij}) = \sum_{i \in N} (y_{ji} - t_{ji} x_{ji}) \qquad j \in N \setminus \{0\} \tag{1e}$$

$$y_{j0} \leq T x_{j0} \qquad j \in N \setminus \{0\} \tag{1f}$$

$$y_{ij} + w_{ij} \leq (T - d_j - min_l\{t_{jl}\}) x_{ij} \qquad i \in N, j \in N \setminus \{0\}, i \neq j \tag{1g}$$

$$\sum_{i \in N} (y_{ij} + w_{ij}) \leq b_j - d_j \qquad j \in N^s \tag{1h}$$

$$\sum_{i \in N} (y_{ij} + w_{ij}) \geq a_j \qquad j \in N^s \tag{1i}$$

$$\sum_{i \in N} (y_{ik^s} + w_{ik^s}) + d_k \leq \sum_{i \in N} (y_{il^g} + w_{il^g}) + M(1 - u_{k^s l^g}) \qquad k^s \in N^s, l^g \in N^g, k = l \tag{1j}$$

$$\sum_{i \in N} (y_{il^g} + w_{il^g}) + d_l \leq \sum_{i \in N} (y_{ik^s} + w_{ik^s}) + M u_{k^s l^g} \qquad k^s \in N^s, l^g \in N^g, k = l \tag{1k}$$

$$\sum_{j \in N \setminus \{0\}} x_{0j} \leq V \tag{1l}$$

$$x_{ij} \in \{0,1\}, y_{ij}, w_{ij} \geq 0 \qquad i, j \in N, i \neq j \tag{1m}$$

$$u_{i^s, j^g} \in \{0,1\} \qquad i^s \in N^s, j^g \in N^g, i = j \tag{1n}$$

Observe that the time is modelled as a flow system whose source and sink are node 0, where the flow values increase as patients are visited and cared. (1b) and (1c) ensure that all the tasks are executed; (1d) and (1e) ensure the connectivity of the routes and time continuity; (1f) and (1g) ensure that the duration of a route does no exceed T and together with (1b), (1c), (1d) and (1e) prevent the presence of sub-circuits; (1h) and (1i) ensure the that the time windows are satisfied; (1j) and (1k) prevent that patients are treated for two different needs at the same time; (1l) ensure that the total number of vehicles is not exceeded and, finally, (1m) and (1n) define the variable domain.

In the talk, we introduce a set of valid inequalities and empirically evaluate the model.