On blocking the spread of harmful contagions in networks with integer programming*

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Extended Abstract

The COVID-19 pandemic is a somber reminder of the danger of the spread of harmful contagions in networks. It has infected over 250 million people and caused the death of over 5 million people[1]. The pandemic also caused serious economic damage, see, e.g., [11]. Moreover, the spread of harmful contagions is not only restricted to the spread of an infectious disease, but can also occur as computer viruses and malware in computer networks [15]. Furthermore, the spread of fake news and propaganda in online social networks is also a very pressing issue [10].

In this work, we introduce the *measure-based spread minimization problem*, which can be used to model how to optimally minimize the spread of harmful contagions in networks. We are given a directed graph $G = (V, A)$ representing a network, a stochastic diffusion model for spread in the network, and a set of initially infected nodes $I \subset V$. Let K be a set of labels each of which represents a certain relationship (contact) type. While there can be multiple arcs between a node pair, each arc is labeled with exactly one label. Blocking a label means taking a measure that prevents the contact between every pair of nodes connected via an arc having that label. In other words, there is a measure associated with each label which causes the arcs with the relevant label to be removed from the network. In a disease spread context, possible measures could be closing of schools, closing of department stores, and the lock-down of a certain area. A formal definition of the problem is given as follows.

Definition 1 (Measure-based spread minimization problem)**.** *Let $G = (V, A)$ be a graph representing a network and K be a finite set of labels. For each $k \in K$, we are given a* measure cost $c_k \geq 0$. *We are given a label $\ell(i, j) \in K$ for each $(i, j) \in A$, a set of initially infected nodes $I \subset V$, and a stochastic diffusion model $\mathcal{M}$. Let $\sigma_{\mathcal{M}}(G, I)$ denote the expected number of infected nodes due to a spread triggered by the seed set $I \subset V$ on network G, under a stochastic diffusion model $\mathcal{M}$. The* measure-based spread minimization problem *consists of finding a set of measures to take (labels to block) within a budget such that the expected number of infected nodes is minimized. Formally it is defined as*

$$\min_{K' \subset K : c(K') \leq B} \sigma_{\mathcal{M}}(G_{K'}, I)$$

where $G_{K'} = (V, A \setminus \{(i, j) \in A : \ell(i, j) \in K'\})$ and $c(K') \leq B$ is the budget constraint.

In [9] spread-blocking problems were introduced considering a deterministic diffusion model and an objective of saving all non-seed nodes from infection while minimizing the cost for the needed blocking-actions. Simple heuristics to solve them were presented. Similar heuristic work for spread-blocking problems based on edge/node deletion in deterministic networks is also done in [8, 3, 7].

There exist also some work using exact methods to tackle problems related to ours: In [4], the authors study which arcs to remove to minimize the spread in a deterministic linear threshold model and present integer programming (IP) approaches for their problem. Another deterministic node-deletion problem

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[1]according to `https://covid19.who.int/`, 15th November 2021

motivated by the spread of influenza-virus was studied in [2]. In [5] a robust version of a node-deletion problem is considered and IP approaches for solving it are presented. In [13] the goal is not to directly minimizing the contagion but to reduce connectivity of the network by node deletions. A similar problem is also tackled with IP and heuristics in [1]. In [12] two "spread related" deterministic metrics are defined and IP formulations are developed to optimize these metrics through link removal. In [14] a bilevel stochastic spread-blocking problem based on node-deletion is considered and solved using IP.

We present IP approaches to model our problem. The modeling approach is based on *stochastic programming* and *live-arc* graphs (see, e.g., [6, 14]) to model the diffusion process. We propose a Benders decomposition based solution algorithm to allow for the solution of large-scale instances. The algorithm is enhanced with various components. We present a computational study to analyse the effectiveness of our algorithm and also to gain managerial insights.

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