Short Paper

Characterizing Path-Length Matrices of Unrooted Binary Trees

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Abstract We address a long standing open question concerning the existence of sufficient conditions that a $n \times n$ symmetric integer matrix must satisfy to encode the *Path-Length Matrix* (PLM) of an Unrooted Binary Tree (UBT) with n leaves. This question is central in the applications of matrix fitting as well as in the combinatorics of the *Balanced Minimum Evolution Problem* (BMEP), an $\mathcal{APX}$-hard network design problem having fundamental applications in epidemiology and life sciences. We show here that Kraft's equalities and Buneman's four-point conditions, in addition to the symmetry and integrality conditions on the entries of a candidate matrix, are necessary and sufficient to ensure that it encodes the PLM of an UBT. We also show that alternative characterizations are possible by exploiting the combinatorics of UBTs. These results provide insights on the polyhedral combinatorics of the BMEP and enable the development of integer linear programming formulations for the problem able to solve instances that are currently intractable via the current exact solution algorithms.

Keywords: *Combinatorial Optimization, Integer Programming, Polyhedral Combinatorics, Network Design, Tree Fitting, Matrix Fitting.*