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# On Minimally Non-Firm Binary Matrices

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## Abstract

For a binary matrix  $X$ , the Boolean rank  $\text{br}(X)$  is the smallest integer  $k$  for which  $X$  equals the Boolean sum of  $k$  rank-1 binary matrices, and the isolation number  $i(X)$  is the maximum number of 1s no two of which are in a same row, column and a  $2 \times 2$  submatrix of all 1s. We continue Lubiw's study of firm matrices,  $X$  is firm if  $i(X) = \text{br}(X)$  and this equality holds for all its submatrices. We show that a stronger concept of superfirmness of  $X$  is equivalent to having no odd holes in a graph defined from  $X$ . Then we introduce two matrix operations that lead to generalised binary matrices and use them to derive four infinite classes of minimally non-firm matrices, matrices which are not firm but all of their proper submatrices are.

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